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On the effect of friction in steady flow of dense gases in pipes

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Abstract

The effect of friction in steady, one-dimensional flow of a real gas in pipes is treated in the paper, with an emphasis on the dense gas effects. In addition to the well-known fundamental derivative of gas-dynamics Γ, another derivative, defined as $\Gamma_1 = 1 + \rho(\partial c/\partial \rho)_T/c$, is shown to play a very important role in studying these non-isentropic flows. In both of two characteristic cases of flow, isothermal and adiabatic one, pressure and density variations along the pipe are not affected by the dense gas effect, their variations being qualitatively the same as in perfect gas theory. Variations of the temperature and of the Mach number are, however, severely influenced by the dense gas effect. Perhaps the most important result from the practical point of view is that isothermal flow in the $\Gamma_1 < 0$ region is choking-free, and that adiabatic flow in the $\Gamma < 0$ is almost choking-free. Analytic considerations derived in the paper are supported by a number of numerical results for a van der Waals gas.

Keywords: Dense gases; Effect of friction in pipes; Isothermal flow; Adiabatic flow

1. Introduction

Although most flow phenomena in gas-dynamics can be accurately enough characterized by using the model of a perfect gas, there are several phenomena of great practical importance which require some refined gas models for their description. Such flow phenomena usually take place in the vicinity of the saturated vapor line, in the single-phase regime of flow, and are governed by one of the several equations of state existing in the literature, like van der Waals, Redlich–Kwong, Martin–Hou equations of state, and others. Among them gases for which the quantity:

$$\Gamma = 1 + \frac{\rho}{c}\left(\frac{\partial c}{\partial \rho}\right)_s, \tag{1}$$

may attain negative values constitute a very important class of fluids which under certain conditions exhibit very unexpected and peculiar behavior in that the most flow properties of perfect gases are inverted. Such a behavior is shown to be inherent in both isentropic and non-isentropic flows, and the most striking examples are the non-monotone variation of the Mach number with density in isentropic nozzle flow, and the appearance of expansion shocks, the partial disintegration of both compression and expansion shocks, shock splitting, etc., in non-isentropic flow. That is why the quantity Γ is given the name fundamental derivative of gas-dynamics (see Thompson, 1971), while the gases with $\Gamma < 0$ behavior are given several names in the literature, like: Bethe–Zel'dovich–Thompson (BZT) fluids, fluids with negative non-linearity, and dense gases.

Given a concrete equation of state, Γ can readily be evaluated from its definition (1). For a perfect gas Γ is positive and equal $(\gamma + 1)/2 = \text{const.}$, while for real gases it is in general a function of two arbitrarily chosen state variables. Calculations performed in the (p, V) plane show that $\Gamma < 0$ behavior can be exhibited by gases which are characterized by relatively high values of the ratio $c_{v0}^{(c)}/R$ only, typical minimum value of this ratio being about 17 for a van der Waals gas. So high values of this ratio are expected to be observed in gases of relatively complex molecular structure, and, indeed, calculations with most hydrocarbons and fluorocarbons performed by using several equations of state, *s.* Lambrakis and Thompson (1972) and Cramer (1989), clearly show the existence of a $\Gamma < 0$ region in the (p, V) plane for these gases. This region is a tongue-shaped one and extends over a general neighborhood of the saturated vapor line at pressures and temperatures of the order of their critical values.

As far as the non-isentropic flows of dense gases are concerned, the most of the literature until now has been addressed to the study of shocks (see for e.g., Thompson and Lambrakis, 1973; Cramer and Sen, 1986). However, the other non-isentropic flows, like those influenced by the friction and/or the heat exchanged with the environment, are not less important from the practical point of view. Several engineering problems which include the flow of a real gas in the dense gas regime,

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Notation

c	speed of sound
c_p	constant pressure specific heat
c_v	constant volume specific heat
c_{v0}	constant volume specific heat at zero density
$c_{v0}^{(c)}$	constant volume specific heat at zero density and critical temperature
D	pipe diameter
e	internal energy
M	Mach number
p	pressure
p_r	reduced pressure
δq	heat exchanged with environment on the distance dx along the pipe
R	gas constant
s	entropy
T	temperature
T_c	critical temperature
T_r	reduced temperature
v	fluid velocity
$V = 1/\rho$	specific volume
x	coordinate in the direction of flow
Greeks	
γ	ratio of specific heats
Γ, Γ_1	fundamental derivatives
λ	friction coefficients
ρ	density
ρ_c	critical density

like the transport in relatively long pipe lines, chemical transport and processing, flow in some parts of power systems equipment, etc. belong to such non-isentropic flows. According to our knowledge, only adiabatic (Fanno) flow of dense gases in long straight pipes has been touched upon in the literature until now. A purely qualitative analysis of this flow is given in Thompson (1971), emphasizing the role of the $\Gamma < 0$ regime in Fanno flow.

In this paper, we analyze compressible steady flow of a real gas in straight pipes affected by friction into more details, with the emphasis on the dense gas effect. In addition to adiabatic flow we treat isothermal flow too, and show that another derivative, denoted in the paper by Γ_1 and defined as:

$$\Gamma_1 = 1 + \frac{\rho}{c}\left(\frac{\partial c}{\partial \rho}\right)_T \tag{2}$$

also plays an important role in studying the effect of friction on the flow of dense gases in pipes. For a perfect gas $\Gamma_1 = 1$. We support our analysis with the number of numerical examples, based on the employment of the van der Waals equation.

2. Problem statement and the governing equations

We consider the problem shown in Fig. 1 in which a real gas flows in the direction of x through a long straight constant-area circular pipe of the diameter D. The flow is supposed to be steady and one-dimensional, and affected by the friction and/or the heat exchanged with the environment. Equations governing such a flow are, as usual, a general equation of state written in the form: $p = p(\rho, T)$, continuity equation, momentum equation and energy equation (First Law of thermodynamics). Written in the differential form they read, respectively:

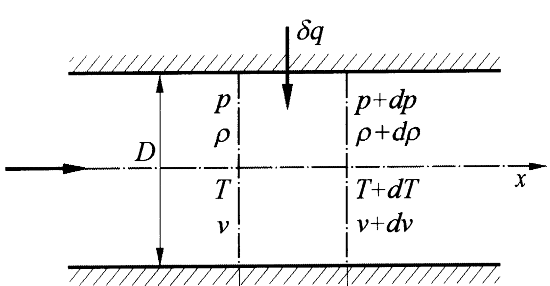

Fig. 1. Real gas flow in a straight pipe.

$$\begin{aligned}
&\mathrm{d}p - \left(\frac{\partial p}{\partial \rho}\right)_T \mathrm{d}\rho - \left(\frac{\partial p}{\partial T}\right)_\rho \mathrm{d}T = 0,\\
&\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}v}{v} = 0,\\
&\frac{\mathrm{d}p}{\rho} + v\mathrm{d}v + \frac{\lambda\,\mathrm{d}x}{2D}v^2 = 0,\\
&\mathrm{d}e - \frac{p}{\rho^2}\mathrm{d}\rho - \frac{\lambda\,\mathrm{d}x}{2D}v^2 - \delta q = 0.
\end{aligned} \tag{3}$$

As well-known (see Moran and Shapiro, 1992), some of the general thermodynamics relations, used here, are:

$$\mathrm{d}e = c_v\,\mathrm{d}T + \frac{1}{\rho^2}\left[p - T\left(\frac{\partial p}{\partial T}\right)_\rho\right]\mathrm{d}\rho, \tag{4}$$

$$c_v = c_{v0} - T\int_0^\rho \left(\frac{\partial^2 p}{\partial T^2}\right)_\rho \frac{\mathrm{d}\rho}{\rho^2}, \tag{5}$$

where $c_v = c_v(\rho, T)$ and $c_{v0} = c_{vo}(T)$. Several empirical formulas for c_{v0} are recommended in the literature. Here we will employ the simplest one which accurately enough describes the variation of c_{v0} in the vicinity of the critical temperature T_c, stated in Thompson and Lambrakis (1973):

$$c_{v0} = c_{v0}^{(c)}\left(\frac{T}{T_c}\right)^n \tag{6}$$

with $n \approx 0.75$ for most hydrocarbons and $n \approx 0.45$ for most fluorocarbons. Also, we will be using the following definitions/expressions for the speed of sound:

$$c^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = \left(\frac{\partial p}{\partial \rho}\right)_T + \frac{T}{c_v\rho^2}\left(\frac{\partial p}{\partial T}\right)_\rho^2 = \gamma\left(\frac{\partial p}{\partial \rho}\right)_T, \tag{7}$$

where $\gamma = c_p(\rho, T)/c_v(\rho, T)$ is the ratio of specific heats, and:

$$c_p = c_v + \frac{T}{\rho^2}\frac{(\partial p/\partial T)_\rho^2}{(\partial p/\partial \rho)_T}. \tag{8}$$

In order to reveal the possible role of the Mach number M in the problem considered, we will now eliminate the velocity v from the system (3) by its relation to M: $v = Mc$, introduce derivatives Γ and Γ_1 defined by (1) and (2), respectively, and employ (4) and (7). We will get:

$$
\begin{aligned}
&\mathrm{d}p - \frac{c^2}{\gamma}\,\mathrm{d}\rho - \left(\frac{\partial p}{\partial T}\right)_\rho \mathrm{d}T = 0,\\
&\Gamma_1 \mathrm{d}\rho + \frac{c_\mathrm{v}\rho^2}{T}\,\frac{\Gamma - \Gamma_1}{(\partial p/\partial T)_\rho}\,\mathrm{d}T + \frac{\rho}{M}\mathrm{d}M = 0,\\
&\mathrm{d}p + M^2c^2(\Gamma_1 - 1)\,\mathrm{d}\rho + M^2c^2\frac{c_\mathrm{v}\rho^2}{T}\,\frac{\Gamma - \Gamma_1}{(\partial p/\partial T)_\rho}\,\mathrm{d}T\\
&\qquad + \rho M c^2\,\mathrm{d}M = -\rho M^2 c^2\frac{\lambda\,\mathrm{d}x}{2D},\\
&c_\mathrm{v}\,\mathrm{d}T - \frac{T}{\rho^2}\left(\frac{\partial p}{\partial T}\right)_\rho \mathrm{d}\rho = \delta q + M^2c^2\frac{\lambda\,\mathrm{d}x}{2D}.
\end{aligned}
\tag{9}
$$

Before we start analyzing the effect of friction in isothermal and adiabatic flow in pipes, we will pay some attention to the sign of the derivative Γ_1. We will show that, like Γ, Γ_1 also changes sign in a general neighborhood of the saturated vapor line, provided $c_{\mathrm{v}0}^{(\mathrm{c})}/R$ is large enough. For that purpose, we will use the model of a van der Waals gas which was shown to be the most favorable one for the dense gas effects to become apparent (see Thompson and Lambrakis, 1973). The equation of state for a van der Waals gas is well known:

$$p = \frac{RT\rho}{1 - b\rho} - a\rho^2,$$

where $a = 9RT_\mathrm{c}/8\rho_\mathrm{c}$ and $b = 1/3\rho_\mathrm{c}$. For such a gas:

$$c^2 = \frac{RT}{(1 - b\rho)^2}\left(1 + \frac{R}{c_\mathrm{v}}\right) - 2a\rho, \quad c_\mathrm{v} = c_{\mathrm{v}0}(T),$$

$$\Gamma_1 = \frac{1}{1 - b\rho}\left[1 + \frac{a\rho}{c^2}(3b\rho - 1)\right].$$

In Fig. 2 we plot $\Gamma_1 = 0$ lines, together with $\Gamma = 0$ lines, in a non-dimensional (p, V) diagram in which p_r, V_r and T_r represent the corresponding values of reduced state variables, for three different values of $c_{\mathrm{v}0}^{(\mathrm{c})}/R$ and $n = 0.5$, see (6). Inside the tongue-shaped region Γ_1 is negative, and this fact will play an important role in studying the isothermal flow case.

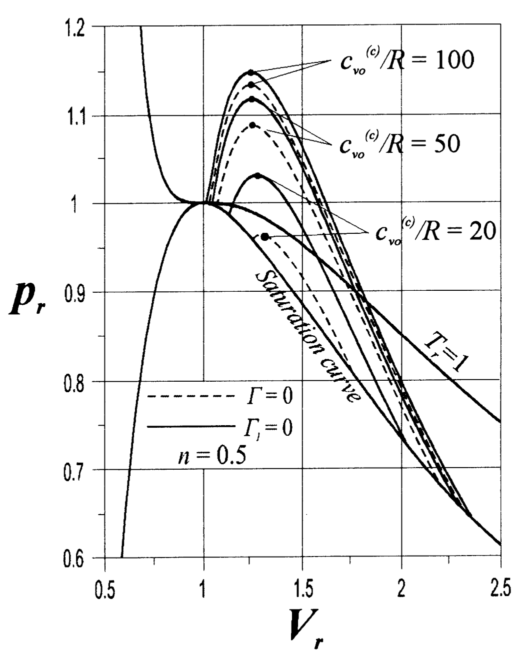

Fig. 2. $\Gamma = 0$ and $\Gamma_1 = 0$ lines in $(p_\mathrm{r}, V_\mathrm{r})$ diagram for a van der Waals gas.

Also we will evaluate the difference $\Gamma - \Gamma_1$ present in (9), for a van der Waals gas:

$$\Gamma - \Gamma_1 = \frac{R^2T}{2c^2c_{\mathrm{v}0}(1 - b\rho)^3}\left[1 + (1 - n)\frac{R}{c_{\mathrm{v}0}}\right].$$

We will note in passing that as long as $0 \leqslant n \leqslant 1$, as for most real gases, $\Gamma - \Gamma_1 > 0$, and that for $c_{\mathrm{v}0}^{(\mathrm{c})}/R \to \infty, \Gamma \to \Gamma_1$, which is clearly indicated in Fig. 2.

3. Isothermal flow

In the isothermal flow case $\mathrm{d}T = 0$, the energy equation in (9) becomes uncoupled from the others and serves for the determination of the heat δq which is spontaneously exchanged with the environment in this case. From the first three equations, we may readily get the following relations suitable for a qualitative analysis of flow:

$$
\begin{aligned}
&\frac{2D}{\lambda}\frac{\mathrm{d}\rho}{\mathrm{d}x} = \frac{\gamma\rho M^2}{\gamma M^2 - 1}, \quad \frac{2D}{\lambda}\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{\rho c^2 M^2}{\gamma M^2 - 1},\\
&\frac{2D}{\lambda}\frac{\mathrm{d}M}{\mathrm{d}x} = -\frac{\gamma\Gamma_1 M^3}{\gamma M^2 - 1}, \quad \frac{1}{M}\frac{\mathrm{d}M}{\mathrm{d}x} = -\frac{\Gamma_1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}x}.
\end{aligned}
\tag{10}
$$

As in the classical case of a perfect gas flow (see Shapiro, 1953), p and ρ decrease in the direction of flow if $M < 1/\sqrt{\gamma}$ ("subsonic" flow), and increase if $M > 1/\sqrt{\gamma}$ ("supersonic" flow). However, the behavior of the Mach number is reminiscent of the one in perfect gas theory (M always tends to the value of $1/\sqrt{\gamma}$ far downstream) in the region $\Gamma_1 > 0$ only. In the dense gas region $\Gamma_1 < 0$ Mach number decreases in a subsonic flow, and increases in a supersonic one, thus showing a non-monotone variation with the density, as in the case of a dense gas isentropic nozzle flow (see Cramer and Best, 1991). This is clearly seen from the fourth of relations (10). Of course γ is not a constant in real gas flows. For a van der Waals gas it can be readily evaluated from (5) and (8):

$$\gamma = 1 + \frac{R^2T}{c_\mathrm{v}\left[RT - 2a\rho(1 - b\rho)^2\right]}.$$

It seems at the first sight that for $c_{\mathrm{v}0}^{(\mathrm{c})}/R$ large enough, γ will be very close to 1 throughout. However, by approaching the critical point in a (p, V) diagram along the critical isotherm γ tends to infinity and experiences very large variations just as in the $\Gamma_1 < 0$ region. These variations are such that γ increases/decreases when ρ increases/decreases. Consequently, M and $1/\sqrt{\gamma}$ variations along the pipe in the dense gas regime of flow diverge in both subsonic and supersonic flow, and the possible cross-section of the pipe in which $M = 1/\sqrt{\gamma}$ is not existent. Thus, choking of an isothermal flow in the pipe in the dense gas regime of flow cannot occur and the mass flow rate through the pipe in this case of flow is not limited. In a supersonic flow there is a tendency toward condensation downstream.

These statements will now be illustrated on a couple of numerical examples. For the need of numerical integration of the system of equations (9) they were first written in non-dimensional form by introducing reduced values of state variables, and were subsequently integrated by the fourth order Runge–Kutta method. At that, the friction coefficient λ was assumed constant. In Fig. 3, variations of Mach number and reduced pressure along the pipe in isothermal subsonic flow are plotted. The values of state variables at the entrance of the pipe were chosen in the $\Gamma_1 < 0$ region, and the integration was performed up to the critical cross-section of the pipe in the $\Gamma_1 > 0$ region, in which choking occurs. Noteworthy is a sharp

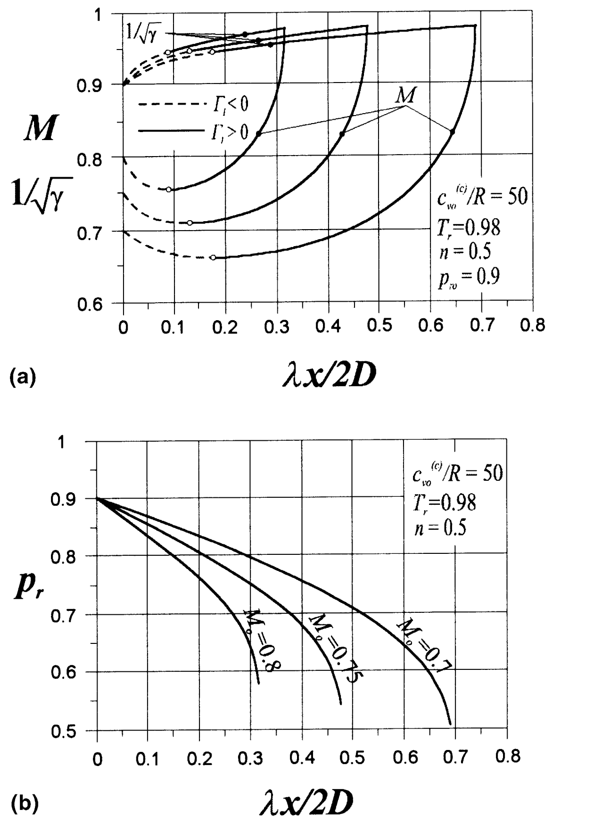

Fig. 3. (a) Mach number variation along the pipe in an isothermal subsonic flow. (b) Reduced pressure variation along the pipe in an isothermal subsonic flow.

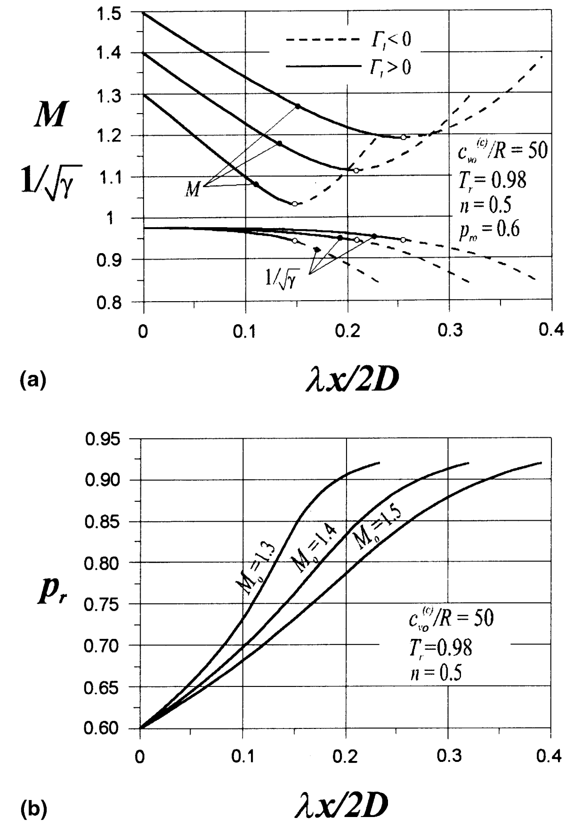

Fig. 4. (a) Mach number variation along the pipe in an isothermal supersonic flow. (b) Reduced pressure variation along the pipe in an isothermal supersonic flow.

dependence of the critical length of the pipe on the initial value of the Mach number. In Fig. 4, variations of the same quantities are plotted for a supersonic flow. Initial values of the Mach number are chosen to be high enough, so that choking does not happen in the $\Gamma_1 > 0$ region, and the integration was performed up to the saturation curve.

Finally, we will conclude our discussion of isothermal flow case with an analysis of the direction in which the heat is exchanged with the environment. By the help of (7) and (10) the energy equation in (9) can be written as:

$$\delta q = -\gamma M^2 \left[\left(\frac{\partial p}{\partial \rho} \right)_T + \frac{T}{\rho} \left(\frac{\partial p}{\partial T} \right)_\rho \frac{1}{\gamma M^2 - 1} \right] \frac{\lambda \, \mathrm{d}x}{2D}. \quad (11)$$

Since both partial derivatives in the bracket are always positive, obviously a supersonic flow is heat releasing, as in the case of a perfect gas (see Shapiro, 1953). However, the sign of δq in a subsonic flow cannot be easily revealed from (11). Careful examination of van der Waals gas shows that δq may switch between positive and negative values in this case. As a rule, low Mach number, high temperature and high density flows are heat releasing, in contrast to perfect gas theory, while the other subsonic flows are heat absorbing. The former conclusion can be inferred from (11) in the following way. At high temperatures and densities isotherms are very steep in the (p, V) diagram, so that the first term in the bracket may prevail over the second one, provided the Mach number is low enough.

Note that all conclusions concerned with δq represent general properties of a van der Waals gas and are not specifically related to the previously discussed dense gas effect.

4. Fanno flow

In adiabatic flow case $\delta q = 0$. Then the following relations suitable for a qualitative analysis of density, pressure, temperature and Mach number variations along the pipe can be derived from (9):

$$\begin{aligned}
\frac{2D}{\lambda}\frac{\mathrm{d}\rho}{\mathrm{d}x} &= -\frac{\rho M^2}{1 - M^2}\left[1 + \frac{1}{\rho c_\mathrm{v}}\left(\frac{\partial p}{\partial T}\right)_\rho\right], \\
\frac{2D}{\lambda}\frac{\mathrm{d}p}{\mathrm{d}x} &= -\frac{\rho M^2 c^2}{1 - M^2}\left[1 + \frac{M^2}{\rho c_\mathrm{v}}\left(\frac{\partial p}{\partial T}\right)_\rho\right], \\
\frac{2D}{\lambda}\frac{\mathrm{d}T}{\mathrm{d}x} &= -\frac{M^2(\gamma M^2 - 1)}{c_\mathrm{v}(1 - M^2)}\left[\left(\frac{\partial p}{\partial \rho}\right)_T + \frac{T}{\rho}\left(\frac{\partial p}{\partial T}\right)_\rho \frac{1}{\gamma M^2 - 1}\right], \\
\frac{2D}{\lambda}\frac{\mathrm{d}M}{\mathrm{d}x} &= M^3\left\{\frac{\Gamma}{1 - M^2}\left[1 + \frac{1}{\rho c_\mathrm{v}}\left(\frac{\partial p}{\partial T}\right)_\rho\right] - \frac{\rho c^2}{T}\frac{\Gamma - \Gamma_1}{(\partial p/\partial T)_\rho}\right\}
\end{aligned} \quad (12)$$

.

Thus, as in a perfect gas flow (see Shapiro, 1953), density and pressure decrease in a subsonic flow and increase in a supersonic one. The temperature also increases in a supersonic flow, while it decreases for $1/\sqrt{\gamma} \leqslant M < 1$. For other subsonic flows the temperature variations cannot be easily deduced from (12).

Interestingly enough, the third expression of (12) can be, by use of (11), written in the form:

$$c_{\rm p}{\rm d}T = \frac{\gamma M^2 - 1}{1 - M^2}\delta q\bigg|_{T={\rm const.}}.$$

Since $\delta q|_{T={\rm const.}} < 0$ for low Mach number, high temperature and high density flow, as deduced earlier, the temperature will increase in such a flow, which is in contrast with perfect gas theory.

It is to be noted that the dense gas effects expressed by means of the derivatives Γ and Γ_1 are present purely in the fourth of expression (12), and consequently they affect the Mach number variation only, exactly as in isothermal flow case. Since in addition to the partial derivatives present in this expression, $\Gamma - \Gamma_1$ is also always positive, as shown earlier, we can deduce the following. If Γ and $1 - M^2$ are of the different sign ($\Gamma > 0$ and $M > 1$ or $\Gamma < 0$ and $M < 1$) Mach number decreases in the direction of flow. Thus, a supersonic flow will be exposed to choking in the $\Gamma > 0$ region, as in the perfect gas theory (see Shapiro, 1953), while a subsonic flow will not experience choking in the $\Gamma < 0$ region. As for the other cases of interest, we can only generally discuss a transonic flow in which the first term on the right of the fourth expression of (12), if it is positive, is expected to prevail over the second one. Thus, in the large Mach number subsonic flow Mach number will increase in the $\Gamma > 0$ region (choking!), exactly as in the small Mach number supersonic flow in the $\Gamma < 0$ region (no choking!). These and other conclusions made will be confirmed by numerical examples which follow and which will also provide some more information about the properties of Fanno flow.

In Fig. 5, we plot Mach number variations, and pressure and temperature variations along the pipe in a subsonic Fanno flow. It is seen that, for the same initial values of M and $p_{\rm r}$ as in the corresponding isothermal flow case (see Fig. 3), these variations proceed very similarly. Critical lengths of the pipe are a little longer than in isothermal flow. Mach number variations in a supersonic flow, plotted in Fig. 6(a), are however considerably different form their counterparts in isothermal flow (see Fig. 4(a)). For the initial Mach number $M_0 = 1.3$ choking occurs in the $\Gamma > 0$ region over a relatively small distance along the pipe. For $M_0 = 1.4$ and $M_0 = 1.5$ choking does not take place in the $\Gamma > 0$. In the $\Gamma < 0$ region, Mach number increases. Pressure and temperature increase too, so that the process does not terminate on the saturation curve. It continues on the left of the tongue-shaped $\Gamma < 0$ region in the (p, V) plane, where the choking occurs before it reaches the shock-condensation curve. In these two cases, critical lengths of the pipe are considerably shifted downstream.

Considering the Mach number variations in Figs. 5(a) and 6(a) one may conclude that the extremum values of M lie exactly on the $\Gamma = 0$ line. This is only approximately true for very large values of $c_{\rm v0}^{(c)}/R$ for which the second term in the bracket on the right-hand side of the fourth expression in (12) is negligibly small. In general, however, for $\Gamma = 0, {\rm d}M/{\rm d}x$ is negative, because $\Gamma - \Gamma_1 > 0$, as shown already. This conclusion contradicts Thompson's (1971) argument that "if $\Gamma > 0$, the effect of friction is to drive the Mach number toward unity; if $\Gamma < 0$, the effect of friction is to drive the Mach number away from unity". To confirm that, we choose a moderately high value for $c_{\rm v0}^{(c)}/R$ of 20 and plot the Mach number variation along the pipe for a subsonic and a supersonic flow in Fig. 7.

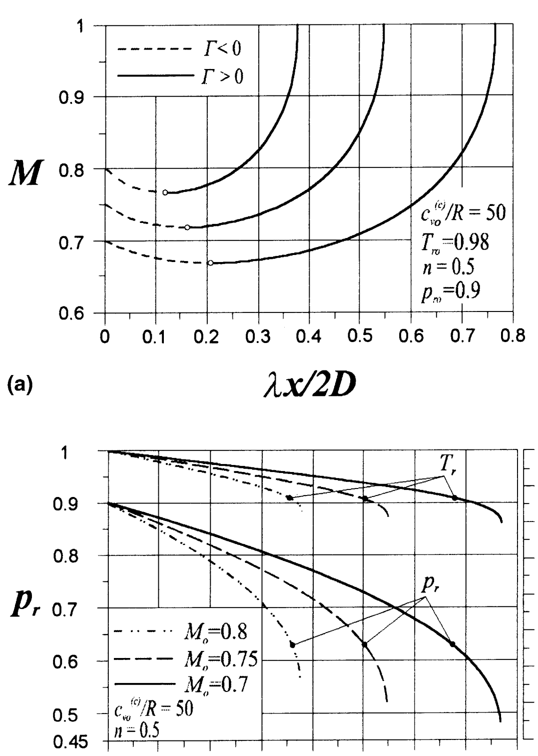

Fig. 5. (a) Mach number variation along the pipe in an adiabatic subsonic flow. (b) Reduced pressure and reduced temperature variations along the pipe in an adiabatic subsonic flow.

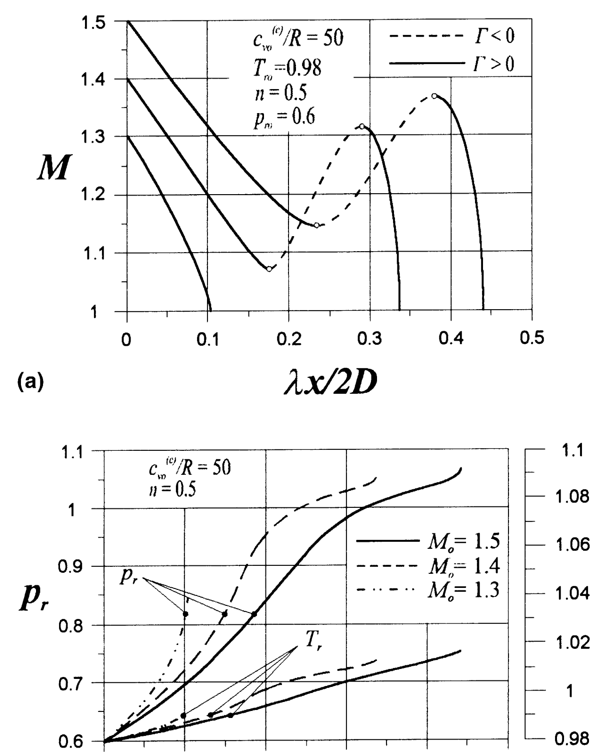

Fig. 6. (a) Mach number variation along the pipe in an adiabatic supersonic flow. (b) Reduced pressure and reduced temperature variations along the pipe in an adiabatic supersonic flow.

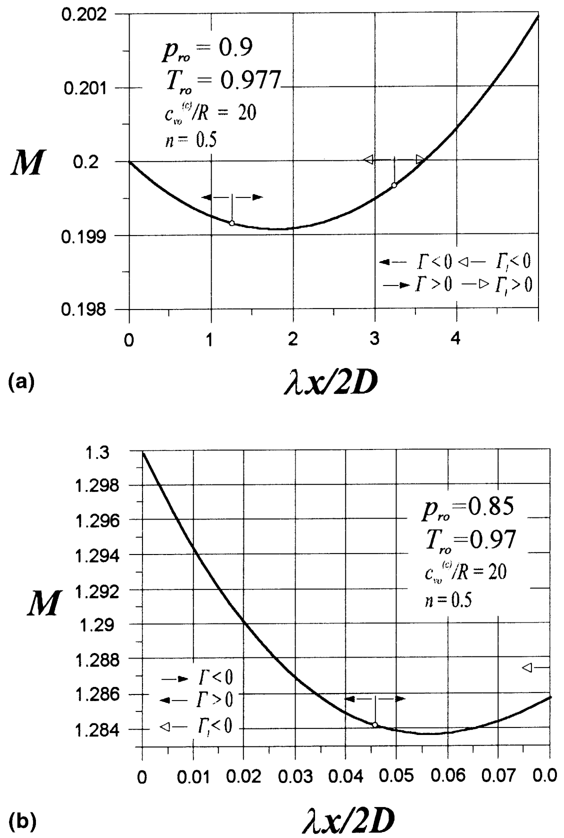

Fig. 7. (a) Mach number variation along the pipe in the vicinity of $\Gamma = 0$ point in an adiabatic subsonic flow for $c_{v0}^{(c)}/R = 20$. (b) Mach number variation along the pipe in the vicinity of $\Gamma = 0$ point in an adiabatic supersonic flow for $c_{v0}^{(c)}/R = 20$.

Clearly, Mach number attains its minimum value in the $\Gamma > 0$ region in the subsonic flow, while this value is located in the $\Gamma < 0$ region in the supersonic flow. Thus, contrary to Thompson's (1971) prediction, there exists a region $\Gamma > 0$, adjacent to $\Gamma = 0$ line, in which the Mach number in a subsonic flow is driven away from unity. Also, there exists a region $\Gamma < 0$, adjacent to $\Gamma = 0$ line, in which the Mach number in a supersonic flow is driven toward unity. Obviously, in the latter case of flow choking may occur in the $\Gamma < 0$ region, while in the former one choking-free area is extended beyond the $\Gamma < 0$ region. When considering possible practical implications of this effect, however, one should have in mind that the effect is pronounced only for lowest possible values of $c_{v0}^{(c)}/R$ for which the dense gas effect is still apparent. Therefore, it may have only a purely theoretical value.

5. Conclusions

It is shown in the paper that, not only some isentropic flows and non-isentropic flows with the presence of shocks, which has been well-known for some time, but also non-isentropic flows in pipes, affected with friction and/or with heat exchanged with the environment, exhibit unexpected and peculiar behavior in a general neighborhood of the saturated vapor line in the (p, V) plane, provided the ratio $c_{v0}^{(c)}/R$ is high enough. In order to describe this behavior qualitatively as well as quantitatively we introduce, in addition to the well-known fundamental derivative Γ, another derivative of some fundamental importance, denoted here by Γ_1, which plays an important role in studying both isothermal and adiabatic flow of dense gases. As Γ, Γ_1 also changes sign (and becomes negative) in a tongue-shaped region in the neighborhood of the saturated vapor line, thus affecting qualitatively the behavior of flow, in particular the isothermal one.

We show that in both subsonic and supersonic flow pressure and density variations along the pipe are not affected by the dense gas effect, their variations being qualitatively identical to those observed in perfect gas theory. Temperature and Mach number variations are, however, severely affected. Qualitative estimations made in the paper are confirmed on a number of numerical examples for a van der Waals gas. From the practical point of view, the most important results are concerned with the phenomenon of choking. It turns out that choking of an isothermal flow cannot happen in the $\Gamma_1 < 0$ region, and that adiabatic flow is almost choking-free in the $\Gamma < 0$ region. Namely, our analysis reveals that only supersonic adiabatic flow for lowest possible values of the ratio $c_{v0}^{(c)}/R$ for which the dense gas effect is still apparent may experience choking in the $\Gamma < 0$ region – the result that may be, therefore, of some theoretical value only, but it contradicts the Thompson (1971) analysis. Hence, in both isothermal and adiabatic flow in pipes the possible choking, if it is not fully avoidable, can be considerably postponed, and the critical length of the pipe increased if the flow process passes through $\Gamma_1 < 0$ and $\Gamma < 0$, respectively.

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